4. Analysis of Plane Trusses and Frames

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An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it. Two types of engineering structures, plane truss and frame, will be discussed in this lesson.

To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free-body diagrams of individual members or combinations of members.

In this statics class, only *statically determinate* structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration, will be considered.

The analysis of trusses and frames under concentrated loads constitutes a straightforward application of the material developed in the previous two topics, *force systems* and *equilibrium*. 
Examples of Truss Structures
**Truss** – structure consisting of two-force members (represented as pin connected) designed to support loads large in comparison to its weight and applied at joints connecting members.

**Simple trusses – build on triangles**
- simplest stable geometric shape
- add two members and one joint at a time

**Plane Trusses:**
- all members lie in a single plane
- forces parallel to plane of truss, but not “in” the plane can be transmitted via non-coplanar load bearing members
Techniques for Truss Analysis

- **Method of joints** – usually used to determine forces for all members of truss

- **Method of sections** – usually used to determine forces for specific members of truss

- **Determining Zero-force members** – members which do not contribute to the stability of a structure

- **Determining conditions for analysis** – is the system statically determinate?
Method of Joints

Do FBDs of the joints

Forces are concurrent at each joint $\Rightarrow$ no moments, just

$\Sigma F_x = 0 \quad \Sigma F_y = 0$

Procedure:

1. Choose joint with
   - at least one known force
   - at most two unknown forces

2. Draw FBD of the joint
   - draw just the point itself
   - draw all known forces at the point
   - assume all unknown forces are tension forces and draw
     - positive results $\Rightarrow$ tension
     - negative results $\Rightarrow$ compression
3. Solve for unknown forces by applying equilibrium conditions in $x$ and $y$ directions: $\Sigma F_x = 0 \quad \Sigma F_y = 0$

4. Note: if the force on a member is known at one end, it is also known at the other (since all forces are concurrent and all members are two-force members)

5. Move to new joints and repeat steps 1-3 until all member forces are known
Sample Problem 4.1

Compute the force in each member of the loaded cantilever truss by the method of joints.

**Solution.**

\[
\begin{align*}
[\Sigma M_E = 0] & \quad 5T - 20(5) - 30(10) = 0 \quad T = 80 \text{ kN} \\
[\Sigma F_x = 0] & \quad 80 \cos 30^\circ - E_x = 0 \quad E_x = 69.3 \text{ kN} \\
[\Sigma F_y = 0] & \quad 80 \sin 30^\circ + E_y - 20 - 30 = 0 \quad E_y = 10 \text{ kN}
\end{align*}
\]
Sample Problem 4.1 (conti.)

\[
\begin{align*}
\sum F_y &= 0 \quad 0.866AB - 30 = 0 \quad AB = 34.6 \text{ kN T} & \text{Ans.} \\
\sum F_x &= 0 \quad AC - 0.5(34.6) = 0 \quad AC = 17.32 \text{ kN C} & \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\sum F_y &= 0 \quad 0.866BC - 0.866(34.6) = 0 \quad BC = 34.6 \text{ kN C} & \text{Ans.} \\
\sum F_x &= 0 \quad BD - 2(0.5)(34.6) = 0 \quad BD = 34.6 \text{ kN T} & \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\sum F_y &= 0 \quad 0.866CD - 0.866(34.6) - 20 = 0 \\
& \quad CD = 57.7 \text{ kN T} & \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\sum F_x &= 0 \quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0 \\
& \quad CE = 63.5 \text{ kN C} & \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\sum F_y &= 0 \quad 0.866DE = 10 \quad DE = 11.55 \text{ kN C} & \text{Ans.}
\end{align*}
\]
Do FBDs of sections of truss cut through various members

**Procedure:**

1. **Determine reaction forces external to truss system**
   - Draw FBD of entire truss
   - Note: can find up to 3 unknown reaction forces
   - Use $\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M = 0$ to solve for the reaction forces

2. **Draw a section through the truss cutting no more than 3 members**

3. **Draw an FBD of each section – one on each side of the cut**
   - Show external support reaction forces
   - Assume unknown cut members have tension forces extending from them
4. Solve FBD for one section at a time using:
\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0 \]

- Note: choose pt for moments that isolates one unknown if possible

5. Repeat with as many sections as necessary to find required unknowns
Sample Problem 4.2

Calculate the forces induced in members $KL$, $CL$, and $CB$ by the 20-ton load on the cantilever truss.

Solution.

Summing moments about $L$ requires finding the moment arm $BL = 16 + (26 - 16)/2 = 21$ ft. Thus,

$$[\Sigma M_L = 0] \quad 20(5)(12) - CB(21) = 0 \quad CB = 57.1 \text{ tons } C \quad \text{Ans.}$$

Next we take moments about $C$, which requires a calculation of $\cos \theta$. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos \theta = 12/13$. Therefore,

$$[\Sigma M_C = 0] \quad 20(4)(12) - \frac{12}{13}KL(16) = 0 \quad KL = 65.0 \text{ tons } T \quad \text{Ans.}$$

Finally, we may find $CL$ by a moment sum about $P$, whose distance from $C$ is given by $PC/16 = 24/(26 - 16)$ or $PC = 38.4$ ft. We also need $\beta$, which is given by $\beta = \tan^{-1}(CB/BL) = \tan^{-1}(12/21) = 29.7^\circ$ and $\cos \beta = 0.868$. We now have

$$[\Sigma M_P = 0] \quad 20(48 - 38.4) - CL(0.868)(38.4) = 0$$

$$CL = 5.76 \text{ tons } C \quad \text{Ans.}$$
Sample Problem 4.3

Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.

Solution.

By the analysis of section 1, CJ is obtained from

\[ \sum M_A = 0 \quad 0.707CJ(12) - 10(4) - 10(8) = 0 \quad CJ = 14.14 \text{ kN C} \]

In this equation the moment of CJ is calculated by considering its horizontal and vertical components acting at point J. Equilibrium of moments about J requires

\[ \sum M_J = 0 \quad 0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0 \]

\[ CD = -18.63 \text{ kN} \]

The moment of CD about J is calculated here by considering its two components as acting through D. The minus sign indicates that CD was assigned in the wrong direction.

Hence,

\[ CD = 18.63 \text{ kN C} \]
Sample Problem 4.3 (cont.)

From the free-body diagram of section 2, which now includes the known value of $CJ$, a balance of moments about $G$ is seen to eliminate $DE$ and $JK$. Thus,

$$[\Sigma M_G = 0] \quad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$$

$$DJ = 16.67 \text{ kN} \ T$$

Ans.

Again the moment of $CJ$ is determined from its components considered to be acting at $J$. The answer for $DJ$ is positive, so that the assumed tensile direction is correct.

An alternative approach to the entire problem is to utilize section 1 to determine $CD$ and then use the method of joints applied at $D$ to determine $DJ$. 
Zero Force Members

Usually determined by inspection

**Method of Inspection:**

1. **Two-member truss joints:** both are zero-force members if (a) and (b) are true
   - (a) no external load applied at joint
   - (b) no support reaction occurring at joint

2. **Three-member truss joints:** non-collinear member is zero-force member if (a), (b), and (c) are true
   - (a) no external load applied at joint
   - (b) no support reaction occurring at joint
   - (c) other two members are collinear
Sample Problem 4.4

Identify all zero-force members for the truss structure below.
Sample Problem 4.5

Identify all zero-force members for the truss structure below.
Sample Problem 4.6

Identify all zero-force members for the truss structure below.
Is the system statically determinate?

Count number of two-force members, $m$
Count number of joints, $j$

1. Internally stable without redundancy (statically determinate)  
   $$ m = 2j - 3 $$
2. Internally stable with redundancy (zero-force members? Statically indeterminate?)  
   $$ m > 2j - 3 $$
3. Internally unstable (underconstrained)  
   $$ m < 2j - 3 $$
Sample Problem 4.7

Analysis of Plane Trusses and Frames
**Frame** – a structure in which at least one of its individual members is a *multiforce member*.

**Plane Frames** – all members and forces lie in a single plane with all moments perpendicular to that plane.

If the frame can be solved by using the equations of statics (the equilibrium equations) alone, it is called a statically determinate frame.

If the frames contain more members or supports that are necessary to prevent collapse, then, as in the case of trusses, the problem is statically indeterminate, and the principles of equilibrium, although necessary, are not sufficient for solution.
Examples of Frame Structures
Types of Frames

Analysis of Plane Trusses and Frames

http://www.tekniksipil.org/civil-engineering
Engineering Mechanics - Statics
Force Representation and FBDs
Sample Problem 4/5 (cont.)

Analysis of Plane Trusses and Frames
Sample Problem 4/6

Analysis of Plane Trusses and Frames

Engineering Mechanics - Statics

http://www.tekniksipil.org/civil-engineering

Engineering Mechanics - Statics
Sample Problem 4/6 (cont.)

Analysis of Plane Trusses and Frames

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Engineering Mechanics - Statics

[Diagram of plane truss with forces and labels:]
- $B_y$ with $B_x$ and $30 \text{ lb}$
- $F$ with $3$ and $4$
- $D$ with $50 \text{ lb}$
- $A_y = 60 \text{ lb}$
- $C_y = 100 \text{ lb}$
- $A_x$ and $E$